A note on the compression of air through repeated shock waves

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SUMMARY

The results of calculations of the compression of air by repeated shock waves are compared with the perfect-gas values given by Evans & Evans (1956). The comparison emphasizes the increasing divergence of real from perfect-gas results as shock strengths are raised. The equations relating conditions across a shock wave are obtained in a convenient form for solution using a Mollier diagram.

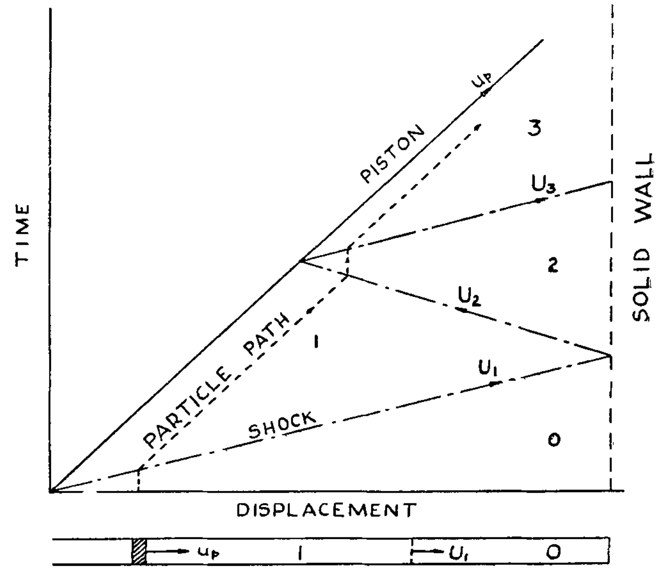

Figure 1. The displacement *vs* time diagram.

Figure 1 shows the problem considered. A piston instantaneously accelerated to a velocity u_p, which is subsequently held constant, generates a shock wave. The wave is reflected from the closed end and the piston face. The air pressure, density and temperature rise non-isentropically through the initial and reflected shocks, soon reaching values at which the excitation of vibrational modes, dissociation, electronic excitation and finally ionization become important. These real-gas effects are taken into account here by using the Mollier diagram of Feldman (1957) prepared from the real-air tables of Hilsenrath & Beckett (1956). All viscous, heat transfer and relaxation effects are neglected and the flow is assumed to be one-dimensional.

The notation is as follows.

- n An integer, referring to a flow region in the time-displacement diagram (figure 1). Thus H_n, ρ_n, p_n and T_n are the enthalpy, density, pressure and temperature in the nth region. The nth shock wave moves into the $(n-1)$th region and has the nth region behind it.
- U_n Velocity of the nth shock wave.
- u_n Flow velocity in the nth region ($u_n = u_p$ for n odd, $u_n = 0$ for n even).
- q Velocity relative to the shock wave.
- M_s Shock strength = (initial shock velocity)/(speed of sound in region 0) = U_1/a_0.
- π A pressure of one atmosphere.

The equations for a stationary shock wave are:

energy $$H_0 + \tfrac{1}{2}q_0^2 = H_1 + \tfrac{1}{2}q_1^2, \tag{1}$$

momentum $$p_0 + \rho_0 q_0^2 = p_1 + \rho_1 q_1^2, \tag{2}$$

continuity $$\rho_0 q_0 = \rho_1 q_1. \tag{3}$$

These may be used for a moving shock wave by superposition of velocities, giving for the initial shock wave

$$q_0 = U_1, \qquad q_1 = U_1 - u_1.$$

Since $u_1 = u_p$, the above three equations yield

$$H_1 - H_0 = \tfrac{1}{2}u_p^2\left(\frac{\rho_1+\rho_0}{\rho_1-\rho_0}\right), \quad p_1 - p_0 = u_p^2\left(\frac{\rho_1\rho_0}{\rho_1-\rho_0}\right), \quad U_1 = \frac{u_p}{1-(\rho_0/\rho_1)}. \tag{4}$$

In the same way the following equations for the first reflected shock are obtained:

$$H_2 - H_1 = \tfrac{1}{2}u_p^2\left(\frac{\rho_2+\rho_1}{\rho_2-\rho_1}\right), \quad p_2 - p_1 = u_p^2\left(\frac{\rho_2\rho_1}{\rho_2-\rho_1}\right), \quad U_2 = \frac{u_p}{(\rho_2/\rho_1)-1}. \tag{5}$$

Similar formulae may be obtained for further shock reflections. The general forms of the equations are:

$$H_{n+1} - H_n = \tfrac{1}{2}u_p^2\left(\frac{\rho_{n+1}+\rho_n}{\rho_{n+1}-\rho_n}\right), \tag{6}$$

$$p_{n+1} - p_n = u_p^2\left(\frac{\rho_{n+1}\rho_n}{\rho_{n+1}-\rho_n}\right), \tag{7}$$

$$U_n = \frac{\mp u_p}{1-(\rho_{n-1}/\rho_n)} \qquad (+ \text{ for } n \text{ odd}, - \text{ for } n \text{ even}). \tag{8}$$

On the Mollier diagram, figure 2, enthalpy is plotted against entropy for lines of constant pressure, lines of constant density and lines of constant temperature. Thus any two of these properties defines a point on the diagram. Given the initial conditions in region 0 and a value for M_s (or piston speed), two values for ρ_1 are chosen. The two corresponding values of H_1 calculated from (4) are plotted on the Mollier diagram, and the points joined by a straight line. The two values for the pressure p_1 from (4) are also plotted, and the line linking these points will cut that joining the enthalpy values if the chosen densities are reasonable. The point of intersection gives the approximate values for conditions in

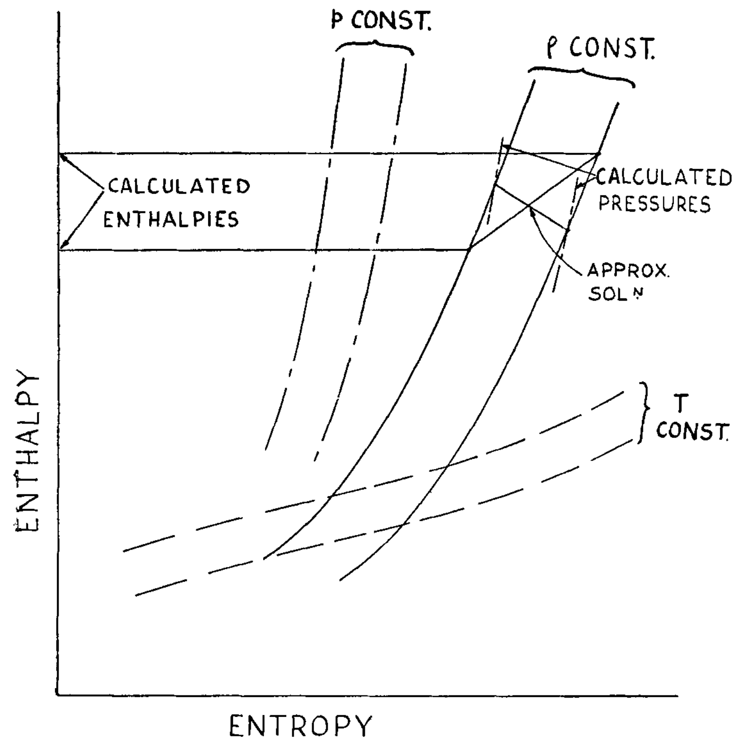

Figure 2. Mollier diagram showing method of solution.

region 1. The approximation may be improved by choosing two new density values just either side of the first intersection point and repeating the construction. With a little experience this becomes unnecessary. Now, knowing conditions in region 1, those in region 2 follow from (5) on using the same graphical construction. Similarly the method can be used for the other regions. The process is then repeated for further values of M_s. A comparison with Feldman's electronically-computed values for region 2 showed a maximum variation of 3%. The values for region 3 are thought to be accurate to 5%.

The results for an initial temperature T_0 of 290° K are shown in figures 3, 4 and 5. Conditions after one, two and three shocks are plotted against initial shock strength and compared with the perfect-gas solutions.

After two reflections only ($n = 3$), the perfect-gas solutions diverge from the air values for initial shock strengths greater than two. Table 1 presents a comparison at various piston speeds.

Thus perfect-gas performance calculations for shock tubes of the multiple reflection type are invalidated and real-gas tables must be used.

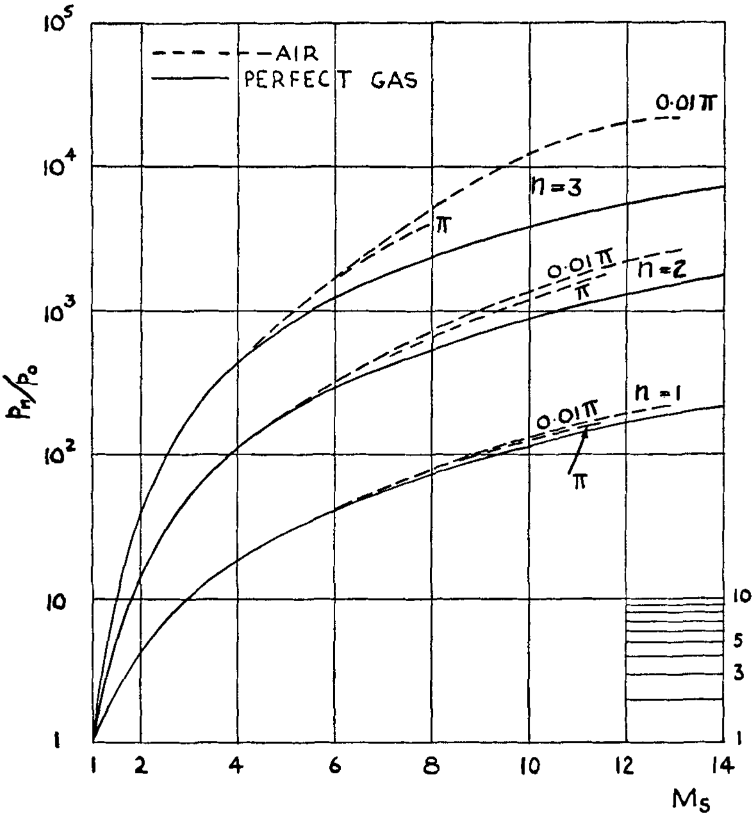

Figure 3. Pressure *vs* shock strength. π and $0{\cdot}01\pi$ refer to the values of p_0 ($\pi = 1$ atmos.). The curve for $p_0 = 0{\cdot}1\pi$ lies between the other two curves for air and has been omitted for the sake of clarity.

Piston speed (ft./sec)	Perfect gas			Air ($p_0 = 0{\cdot}01\pi$)		
	p_3/p_0	T_3/T_0	ρ_3/ρ_0	p_3/p_0	T_3/T_0	ρ_3/ρ_0
2 000	90	5·1	17·5	90	4·8	20·0
4 000	595	15·3	37·0	600	10·6	57·0
6 000	1 500	31·0	46·0	1 900	16·8	107·0
8 000	2 700	53·4	51·0	5 300	23·0	158·0
10 000	4 300	83·0	52·5	11 300	28·2	222·0
12 000	6 100	118·0	53·0	18 900	33·0	292·0

Table 1.

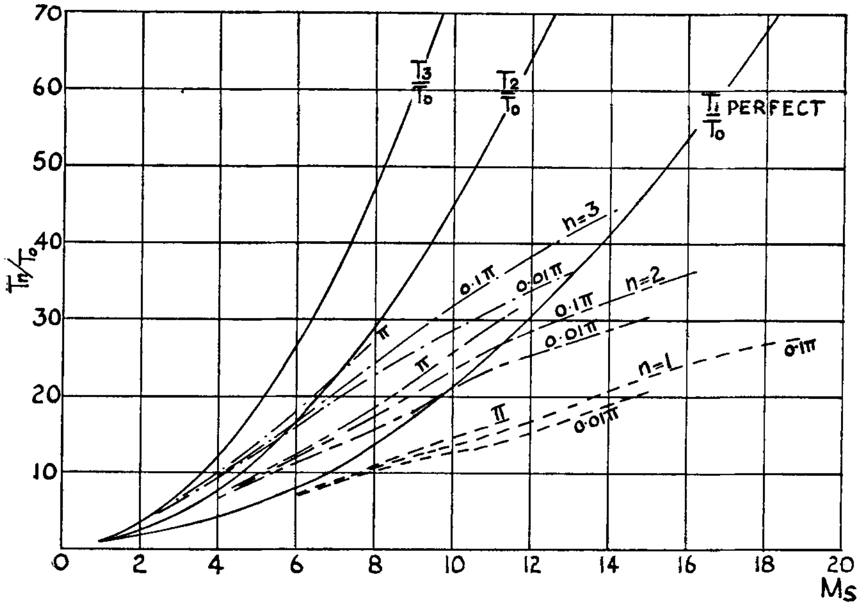

Figure 4. Density *vs* shock strength.

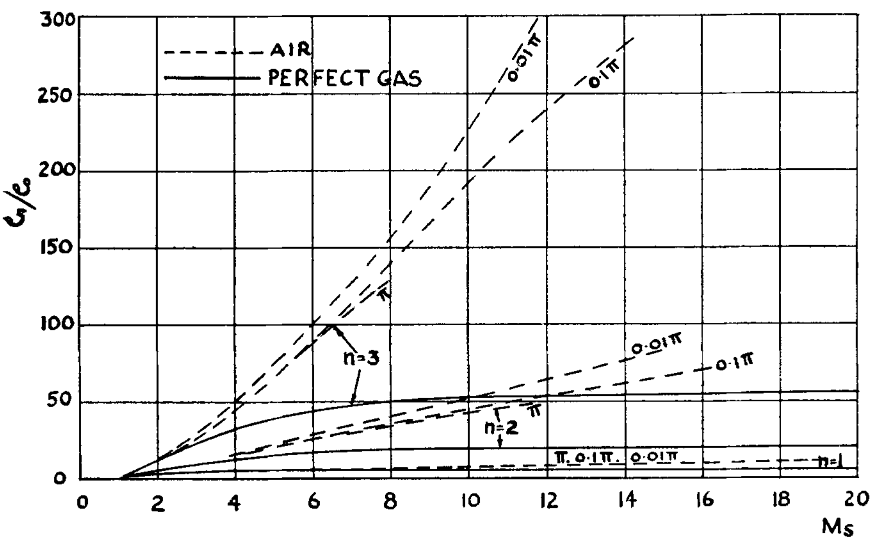

Figure 5. Temperature *vs* shock strength.

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